ON THE VARIABILITY IN GRAVITATIONAL CONSTANT MEASUREMENTS

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INTRODUCTION

Measurements of the Gravitational Constant have large variations both within individual experimental runs and also between results from different experiments. Noise levels in these experiments are temporally and spatially variable and, in certain cases, the source of the noise cannot be located. This situation leads us to consider the possibility that other forces may be affecting these measurements.

Einstein and Cartan envisaged a unified field theory where the electromagnetic field and gravity were related components of a more general geometric model of space-time. So they incorporated torsion as well as asymmetric curvature and metric in this more general space-time. The fundamental equations that relate asymmetric curvature and torsion are the Bianchi Identities, which, for the most general case, are highly non-linear.

We have derived new linearized Bianchi Identities that we find govern solely second-order tensors, Davies (1988). These second order tensors are useful because they can be uniquely separated into independent symmetric and antisymmetric components. By examining a subset, these linearized Bianchi Identities yield the usual relationship of symmetric curvature to the gravitational field gradient. However, an additional component, due to torsion gradients, also contributes to the gravitational field.

The antisymmetric curvature component was discovered to be produced by the divergence of a modified dual of the torsion tensor. The time-like components of the torsion tensor can also be split into symmetric and antisymmetric components both of which, after taking the covariant divergence, yield two independent currents, related to the antisymmetric curvature.

Many researchers, since Einstein, have proposed that torsion terms are related to electromagnetic fields and/or spin. By applying a 'particle-in-field' model of particle-antiparticle pair production, Davies (1988) identified curvature with the massive, charged particles. The forces intermediating the particle and anti-particle are the electroweak fields which were identified with time-like components in the torsion tensor. Using this simple fundamental model, similar equations to Maxwell's for the electromagnetic interactions were found to be incorporated in these new linearised Bianchi Identities.

Our identification of electromagnetic fields with time-like torsion components allows us to examine the consequences especially to measurements of G, the gravitational constant. If torsion (electromagnetic) field gradients are present at the site of the G experiment, the new linearized identity predicts the effect on the observed G values. We examine the results of various old and recent G-measuring experiments and the observed noise levels and value differences between measurements. These are shown to be consistent with the predictions of our unified model and the known electromagnetic field gradients near the surface of the Earth.

RIEMANN-CARTAN SPACE-TIME

The space-time of interest contains torsion as well as curvature and assumes an asymmetric metric tensor. This has been termed a Riemann-Cartan U4 space-time. The fundamental fields for measuring change in a vector around a circuit C in the body of U4 are called the coefficients of affine connection. For general curves C, the closure failure may be caused by both asymmetric curvature and torsion in the space-time where is an asymmetric metric tensor, Schouten (1954).

We examine the case of sufficiently small fields such that linearization is valid and only terms of first order are incorporated. To this order, the connection $L_{\alpha\beta\mu\nu}$ is antisymmetric between the pair of indices $\alpha\beta$ as well as $\mu\nu$. Thus it suffices to consider, to this order, the related second-order asymmetric curvature tensor:

$$(1) \qquad L^{\gamma}_{\delta} = -\frac{1}{4} \cdot \epsilon^{\chi\gamma\alpha\beta} \epsilon_{\chi\delta\mu\nu} \cdot L^{\mu\nu}_{\alpha\beta}$$

whose symmetric part is the usual Einstein tensor, and ϵ is the usual alternating tensor. Because χ is a "dummy" index, this asymmetric curvature tensor can be equivalently expressed as

$$(2) \qquad L^{\gamma}_{\delta} = -\epsilon^{0\alpha\beta\gamma} \epsilon_{0\mu\nu\delta} \cdot L^{\mu\nu}_{\alpha\beta}$$

where 0 represents the time component. Torsion is the antisymmetric part of the connection, where the antisymmetry occurs only in two of the indices :

$$(3) \qquad T_{\alpha\beta\gamma} = \frac{1}{2}\left[L_{\alpha\beta\gamma} - L_{\alpha\gamma\beta} \right]$$

We define the modified dual of the torsion as

$$(4)\quad A_{\alpha\chi\mu} = L_{\alpha}^{\ \beta\gamma}\, \epsilon_{\chi\beta\gamma\mu} = T_{\alpha}^{\ \beta\gamma}\, \epsilon_{\chi\beta\gamma\mu}$$

where A is also known as the dislocation density tensor in the theory of defect-containing media, Lardner (1973).

LINEARIZED IDENTITIES

The first Bianchi Identity was linearized, Davies (1988), by extending the 3 dimensional case, Lardner (1973), and incorporating covariant differentiation in the linearized continuity equation

$$(5)\quad L_{\alpha\beta\gamma\delta} = L_{\alpha\beta\delta,\gamma} - L_{\alpha\beta\gamma,\delta}$$

where the fundamental relation used is

$$(6)\quad L_{\alpha\beta\gamma} = \frac{1}{2}\left[g_{\gamma\alpha,\beta} + g_{\beta\alpha,\gamma} - g_{\beta\gamma,\alpha}\right] + T_{\alpha\beta\gamma} + T_{\beta\gamma\alpha} - T_{\gamma\alpha\beta}$$

On substitution of these equations (5), (6) into the definition of the second order curvature tensor (1) and after some manipulation, Davies (1988), we obtained the new Bianchi's First Identity in the limit of small fields and in terms of these curvature and modified torsion tensors:

$$(7)\quad L_{\gamma}^{\delta} = \epsilon^{\mu\delta\alpha\beta} \cdot A_{\mu\beta\gamma;\alpha} - \frac{1}{2}\cdot \epsilon^{\mu\chi\delta\alpha}\, g_{\chi\gamma}\, A^{\beta}_{\mu\beta;\alpha} - \frac{1}{2}\cdot \epsilon_{\mu\gamma\tau\sigma}\, \epsilon^{\mu\delta\alpha\beta} \cdot g^{\sigma,\tau}_{\beta,\alpha}$$

where the semi-colon implies covariant differentiation. This alternate formulation of the first Identity is complemented by the usual Second Bianchi Identity:

$$(8)\quad L^{\gamma\delta}_{\ ;\delta} = 0$$

where, however, $L^{\gamma\delta}$ is asymmetric in this more general approach.

INDEPENDENT RELATIONS

Davies (1988) uniquely split the second order curvature tensor $L_{\alpha\beta}$ into its symmetric Einstein tensor $G_{\alpha\beta}$ and the antisymmetric components $D_{\alpha\beta}$. From the First Bianchi Identity (7), we obtain the relation governing the symmetric component:

$$(9)\quad \frac{1}{2}\epsilon_{\chi\alpha\gamma\delta}\, \epsilon^{\chi\beta\mu\nu}\, g^{\delta,\gamma}_{\nu,\mu} = -G_{\alpha}^{\beta} + \left[\epsilon^{\chi\beta\mu\nu} A_{\chi\nu\alpha;\mu}\right]_{s}$$

where s indicates the symmetric component is taken. Apart from the torsion dependent term, this equation, on expansion, is equivalent to that derived by Weinberg (1972, eq.7.2.6), using the standard approach based on the assumptions of maximally general metric structure, weak i.e. linear fields, the conservation of symmetric curvature and the nonrelativistic limit of Einstein's field equation giving Newton's Law. This fundamental agreement on the production of the gravitational field by the curvature two-tensor yields added significance to the torsion term in (9). This implies that torsion gradients may also contribute to gravitational fields.

Taking the divergence of Equation (9), Davies (1988) obtained $G_{\mu\nu;\nu}$ = 0, which is the usual conservation of symmetric Einstein curvature. Removing this from the Second Bianchi Identity (8) gives $D_{\mu\nu;\nu}$ = 0 which implies conservation of antisymmetric curvature as well.

Removing (9) from (1) shows that the antisymmetric curvature two-tensor $D_{\alpha\beta}$ is related solely to the torsion gradient term. Multiplying the first Bianchi Identity, (7), with the alternating tensor, we get the fundamental equation

$$A^{\alpha\gamma}_{\mu;\gamma} = -\epsilon_{\mu\gamma\delta\chi} D^{\gamma\delta} g^{\chi\alpha} \tag{10}$$

where only the antisymmetric component of curvature $D^{\gamma\delta}$ is involved.

MAXWELL'S EQUATIONS

Davies (1988) argued that a particle-antiparticle pair is the fundamental mass-containing entity that can be formed from sufficiently energetic photon interaction. The massive charged particle and its equal and opposite anti-particle are represented by asymmetric curvature. The fields that intermediate the particle and its anti-particle must therefore be represented by torsion terms. The concept of mass energy-momentum being represented by angular defect curvature fields has previously been used in the Regge calculus of a skeleton space-time, and extended by Davies (1997) in the quantization of space-time.

The fields through which the particle and anti-particle interact are, in the linear regime, the electromagnetic and weak fields. These fields act along the time-like line between the pair so that one index of the modified dual of the torsion will correspond to the "0" component in this U4 space-time. It is therefore to be expected that the fundamental equations of electromagnetism should be embedded in these Bianchi Identities.

On this basis, we modify the Bianchi Identity subset, eqn.(10), by replacing the directional index μ by the time-like index 0.

$$A^{\alpha\gamma}_{0;\gamma} = -\epsilon_{0\gamma\delta\chi} D^{\gamma\delta} g^{\chi\alpha} \tag{11}$$

Note that this equation would have been obtained if the above derivations had been performed using the second order curvature tensor definition, eqn.(2), which incorporates the time component "0", instead of using the general definition, eqn.(1), which involved the dummy index.

The right-hand side of eqn.(11) contains the antisymmetric curvature two-tensor and can be represented by a current, $I_{o\alpha}$. The dual of the torsion tensor, $A_o^{\alpha\gamma}$, may be split into its independent symmetric component, $W^{\alpha\gamma}$, and antisymmetric component, $F^{\alpha\gamma}$. Equation (11) then splits into two independent relations, namely :

$$F^{\alpha\gamma}_{;\gamma} = J^{\alpha} \tag{12}$$

$$W^{\alpha\gamma}_{;\gamma} = K^{\alpha} \tag{13}$$

where the current term I_o^{α} has been split into the independent currents J^{α} and K^{α}.

Equation (12) is identical to Maxwell's "electric" equation with a current source. Maxwell's magnetic equation,

$$\epsilon^{\alpha\mu\nu\lambda} F_{\mu\nu;\lambda} = 0 \tag{14}$$

follows as a consequence of the antisymmetry of $F_{\mu\nu}$ and the gauge condition. Because of the antisymmetry of $F_{\mu\nu}$, the usual conservation of the electric current is implied, i.e.

$$J^{\alpha}_{;\alpha} = 0 \tag{15}$$

VARIATIONS IN MEASUREMENTS OF THE GRAVITATIONAL CONSTANT

The best measurements of the Gravitational Constant G have an accuracy for the individual experiment of about 1 part in 10,000. This error range is due to the noise recorded by the measuring system. Components of this noise are unlocatable with respect to source and also correlated with external phenomena, Michaelis et al. (1996), sometimes to an excessive level. The range of the older and more recent experimental values is shown in the attached Table 1. The inter-experiment accuracy is, from these results, about 1 part in 1000.

Our unified model shows, from eqn.(9), that torsion, and thus electromagnetic, gradients may also contribute to the gravitational field. As did Weinberg (1972), we take the non-relativistic limit of this equation and get Newton's equation with an added torsion gradient term.

$$\nabla^2 \psi = 4\pi G \rho + \nabla . \underline{E} \tag{16}$$

If torsion gradients of sufficient magnitude are present, measurements of G using mass densities can be affected. Constant and variable torsion gradients may thus manifest as noise and error in measurements. We roughly estimate the relative values of such terms where the ratio R is

$$R = e \nabla . \underline{E} \, / \, 4\pi G \rho \tag{17}$$

where e is the unit electric charge and E the electric field near the apparatus.

The electric field near the surface of the earth is about 1 volt/cm, Feynman (1964), and can vary over centimeter distances. With sharp local topography effects, we can expect a range of electric field gradients of 1 to 10 volt/cm/cm. Using these values and assuming a density of 10 gm/cc, we get a range for R from about 0.5 parts in 10,000 to 0.5 parts in 1000. These lower values are consistent with observed noise values in the quieter individual experiments, the usual ones reported. The higher values are consistent with inter-experiment differences and in the higher levels of noise observed in certain experiments. That a fundamental constant such as G is only measurable to such a poor accuracy is telling us that other forces may be at work.

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VALUE	PRECISION	EXPERIMENTERS
6.71540	0.00056	Michaelis et al., 1996, Metrologia, 32, 267.
6.6873	0.0094	J.P. Schwartz et al., 1998, Science, 282, 2230.
6.6755	0.0008	Heyl, P.R. and P. Chrzanowski, 1942, J.Res.Nat.Bur. Stds., 29, 1.
6.6745	0.0008	Sagitov et al., 1981, Sov.Phys.-Doklady (EarthSci.), 245, 20.
6.6740	0.0007	C.H. Bagley and G.G. Luther, 1997, Phys. Rev. lett., 78, 3047.
6.6726	0.0005	G.G. Luther and W.R. Towler, 1982, Phys. Rev. Lett., 48, 121.
6.6719	0.0008	H. Walesh et al., 1995, IEEE Trans. Instrum. Meas., 44, 491.
6.6714	0.0006	C. Pontikis, 1972, Comptes Rendus Acad. Sci.Ser. B, 274, 437.