CRACKS IN SPACE-TIME

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Abstract

Extrapolation of 3D models of cracks and their mathematical structure leads into a U_4 space-time with torsion and curvature. Creation of a crack in space-time is equivalent to particle - antiparticle pair production. The crack-tips have curvature and correspond to particle currents, the crack-body symbolising the interactive fields. The modified torsion tensor is recognised as the electroweak field, the linear Bianchi identity being equivalent to Maxwell's current equation; the antisymmetric curvature tensor, on contraction, yielding the electroweak current. The linear Bianchi Identity also produces the usual structure of the symmetric Einstein curvature tensor in terms of the metric's derivatives. An additional electroweak contribution is found. "Strong" forces are governed by the non-linear Bianchi Identities. Feynman diagrams are maps of interacting crack-tips and fields. The crack model thus describes the interwoven patterns of the basic currents and fields of physics.

Introduction

Let's first consider a crack (or rip) in a simple 2D medium, for example. Such a crack (see Figure 1) is composed of a body, due to translational displacement, bounded at each end by crack tips due to rotational displacements, Davies (1980). Without this crack the medium would be flat; with it, the medium has both torsion and curvature. The tensor density, curvature, measures the defect angle of the crack tips which are usually termed disclinations. Torsion is the measure of the translational displacement of the crack body and, on contraction, is also known as the dislocation density tensor.

Now, a crack can be created in 3D space by imposing sufficient stress on the medium. We know that sufficient stress of space-time will yield particle-antiparticle pairs which are usually composed of massive, charged, spinning (anti) particles connected by electromagnetic and weak fields. The (anti) particle currents are represented by curvature (crack tips) while the torsion (crack-body) must represent the connecting fields.

By examining the change of a vector taken around a parallelogram enclosing such a crack and the closure failure in this U_4 space-time, the fundamental conservation laws known as the Bianchi Identities can be obtained, c.f. Schouten (1954). These usually relate torsion, a third order tensor density, to curvature, but we shall adapt them to relating the second order dislocation tensor density to the second order disclination tensor density, respectively. The main reason for using second order tensors is that they may be uniquely separated into symmetrical and antisymmetrical components. We shall interpret the antisymmetrical dislocation tensor density as the Electromagnetic field where the linearised Bianchi's Identity gives Maxwell's current equations. Similarly, the symmetrical dislocation component is theorized to be the weak field whose divergence yields the non-conserved weak current. Symmetry breaking by the on-diagonal dislocation component is demonstrated through the case of a crack in a 3D continuum. On decomposition, the electroweak field is found to be propagated by massless and massive/charged particles. When we examine the Bianchi Identities for non-linear interactions, i.e., in intense field strengths, the resultant conservation laws describe the "Strong" fields.

The particle current is shown to be represented by the asymmetric curvature tensor density whose symmetric part is the usual Einstein tensor. The antisymmetric part, on contraction, yields the electroweak current derived, through the Bianchi Identities, as the gradient of the electroweak field. The symmetric Einstein tensor density will be shown, through the linearised Bianchi Identities, to be a particular function of the metric and its derivatives -- a result which previously had only been postulated from alternative methods. An extra electroweak term is also involved and has interesting implications. Feynman diagrams will be seen, literally, to be representations of these space-time cracks, the basic vertex being a crack-tip with emanating fields.

Bianchi Identities

The fundamental fields for measuring change in a vector around a circuit in a U_4 are called the coefficients of connection $L_{\alpha\beta\gamma}$. Consider a closed circuit C in the body and construct a continuous local reference configuration (tetrad) for the points of C. If, upon completing the circuit, the orientation of the reference configuration is rotated from its original orientation then C encloses angular defects represented by the curvature or disclination density tensor. Also, the curve C in its local reference configuration will not, in general, be a closed curve. This closure failure is represented by the dislocation density tensor (torsion) to leading order in the limit as C shrinks to zero. However for general curves C, the closure failure may be caused by both dislocations and disclinations.

We examine the rotation of a vector around the closed path C, such that, for the case of sufficiently small fields, linearisation is valid and only terms of first order are incorporated. To this order, $L_{\alpha\beta\mu\nu}$ is antisymmetric between the pair of indices $(\alpha\beta)$ as well as $(\mu\nu)$. Thus it suffices to consider the related tensor $L^{\gamma}{}_{\delta} = -1/4\varepsilon^{\xi\gamma\alpha\beta}\varepsilon_{\xi\delta\mu\nu}L_{\alpha\beta}{}^{\mu\nu}$ whose symmetric part is the usual Einstein tensor, and ε is the usual alternating tensor. This curvature tensor will, in general, be asymmetric and, because ξ is a "dummy" index, can be equivalently expressed as $L^{\gamma}{}_{\delta} = -1/4\varepsilon^{0\alpha\beta\gamma}\varepsilon_{0\mu\nu\delta}L_{\alpha\beta}{}^{\mu\nu}$. Here, 0 represents the time component, and we use this formulation for compatibility with the following field definition.

Torsion is defined as the antisymmetric part of the connection where $T_{\alpha\beta\gamma} = 1/2[L_{\alpha\beta\gamma} - L_{\alpha\gamma\beta}]$. The dislocation density, in 3D, measures the number of dislocation lines through unit area and is the modified dual of the torsion tensor, Lardner(1973). In spacetime, particle-antiparticle pair production occurs in the expanding Universe which implies the use of comoving coordinates. The fields are thus timelike and we generalise the definition of dislocation density in a U_4 space-time to $A_{0\alpha\mu} = L^{\beta\gamma}{}_{\alpha}\varepsilon_{0\beta\gamma\mu} = T^{\beta\gamma}{}_{\alpha}\varepsilon_{0\beta\gamma\mu}$.

Because of the unique separation of second order tensors into symmetric and antisymmetric components, we shall concern ourselves with $L^{\gamma}{}_{\delta}$ and $A_{0\alpha\beta}$ and the Bianchi Identities governing them. In the 3D case, Lardner (1973), the first Identity is obtained from the linearised continuity equation,

$$L_{\alpha\beta\gamma\delta} = L_{\alpha\beta\delta,\gamma} - L_{\alpha\beta\gamma,\delta}$$

and the relation,

$$L_{\alpha\beta\gamma} = [g_{\gamma\alpha,\beta} + g_{\beta\alpha,\gamma} - g_{\beta\gamma,\alpha}] + T_{\alpha\beta\gamma} + T_{\beta\gamma\alpha} - T_{\gamma\alpha\beta}$$

These linearised equations are also valid in the 4D case, but we must put the fields in a covariant form and also substitute covariant differentiation in order to extend the 3D result. Doing so yields,

$$L_{\gamma}{}^{\delta} = \varepsilon^{0\delta\alpha\beta}A_{0\beta\gamma;\alpha} - \frac{1}{2}\varepsilon^{0\xi\delta\alpha}g_{\xi\gamma}A^{\beta}{}_{0\beta;\alpha} - \varepsilon_{0\gamma\tau\sigma}\varepsilon^{0\delta\alpha\beta}g_{\beta,\alpha}{}^{\sigma,\tau} \qquad (1)$$

This is Bianchi's First Identity in the limit of small fields, where $g_{\sigma\beta}$ is the usual metric tensor and the semi-colon implies covariant differentiation. The second Bianchi Identity is the usual

$$L^{\gamma\delta}{}_{;\delta} = 0 \qquad (2)$$

where, however, $L^{\gamma\delta}$ is asymmetric in this more general approach.

We have previously applied these basic conservation laws to the structure of cracks in 3D space, Davies (1980). Cracks are formed by excessive strain in a region of the continuum and we extend this concept to postulating that particle-antiparticle pair production is equivalent to the formation of a crack in space-time. The crack tips, the disclination field, are measured by the curvature, $L_{\gamma\delta}$, tensor, corresponding to the (anti-)particles. The connecting fields, the dislocation density tensor, are those that intermediate the particle and antiparticle i.e. the electroweak forces.

Electroweak and Strong Forces

The idea of relating the electromagnetic field to torsion originated with Einstein and Cartan c.f. Einstein (1956). However, no exact quantitative relationship has previously been established but if we multiply the first Bianchi Identity (1) by $\varepsilon_{0\gamma\delta\xi}$, after raising and lowering with the metric tensor, the fundamental relation is found :

$$A^{\alpha\gamma}{}_{0;\gamma} = -\varepsilon_{0\gamma\delta\xi}L^{\gamma\delta}g^{\xi\alpha} = J_0{}^{\alpha} \qquad (3)$$

where only the antisymmetric component of $L^{\gamma\delta}$ is involved in the current definition. Now split the dislocation density tensor into its symmetric $W^{\alpha\gamma}$ and antisymmetric $F^{\alpha\gamma}$ components. We postulate that $F^{\alpha\gamma}$ is the usual electromagnetic tensor. From equation (3), and splitting the current $J_0{}^{\alpha}$ into the two components $J^{\alpha}{}_{e}$ and $J^{\alpha}{}_{w}$ we obtain

$$F^{\alpha\gamma}{}_{;\gamma} = J^{\alpha}{}_{e} \qquad (4)$$

$$W^{\alpha\gamma}{}_{;\gamma} = J^{\alpha}{}_{w} \qquad (5)$$

Equation (4) is the first of the usual Maxwell equations, the second being $\varepsilon^{\alpha\mu\nu\lambda}F_{\mu\nu;\lambda} = 0$ which is usually derived from antisymmetry of $F_{\mu\nu}$ and the gauge condition. Because of the antisymmetry of $F^{\mu\nu}$ we have the usual conservation of the electromagnetic current $J_e{}^{\alpha}{}_{;\alpha} = 0$. Gogala (1980) has also argued, from a different approach, that the electromagnetic field must be related to a timelike modification of the torsion.

We postulate that the symmetric component $W_{\mu\nu}$ is the "weak" field. On combining this with the electromagnetic field $F_{\mu\nu}$ we get the electroweak field $A_{0\mu\nu}$, the dislocation density. Now, because of the symmetry of $W^{\mu\nu}$, the current derived from its gradient $J^{\alpha}{}_{w}$ is not conserved, which is a fundamental property of the "weak" current, c.f. Taylor, p.17 (1976). Thus the two currents J_e and J_w combine to give the "electroweak" current, which is the contraction of the antisymmetric component of the curvature tensor. We have thus "unified" these two fields, the electromagnetic and weak, through a physical model i.e., a crack in space-time, independently but complementary of gauge methods. That the weak field produces parity symmetry breaking is illustrated in Figure 1B for the crack in a 3D continuum. From equation (3), when the dislocation density tensor has on-diagonal components the resultant disclinations have a geometry whose spatial symmetry is maximally broken compared to the case where off-diagonal dislocations are involved as in Figure 1A. Rauch (1982) has also found that reflection symmetry is sensitive to torsion.

We now come to the question of the so-called "Strong" forces. The Bianchi Identities used above to describe the electroweak interaction are based on the assumption of fields sufficiently small that only first order terms are used. Obviously, when (anti) particles are so close together that the fields are too intense for this assumption, then the non-linear Bianchi Identities must be used, though these are far from simple relations c.f. Schouten, p.144 (1954).

The main property of these non-linear Bianchi Identities, as compared to the linear relationships (1) and (2), is the appearance of additional terms comprised of the tensor product of the torsion with itself and with curvature. Now, Taylor p.156, (1976), remarks "It is perhaps not impossible that, if properly understood, a Yang-Mills theory might make a satisfactory basis for strong interactions". The main property of such Yang-Mills fields is the appearance of products of the electromagnetic fields with their associated potentials as additional terms in the usual Maxwell Equations. Thus, Yang-Mills type equations may be considered a possible subsystem of these non-linear Bianchi Identities which, we postulate, govern the "Strong" interactions. In fact, recent analyses show that solutions of Yang-Mills equations are also solutions of the modified non-linear Bianchi Identities, c.f. Altamirano and Villarroel, (1981).

Gauge approaches to the electroweak interaction predict that the propagators are both massless and massive. In this crack model, if we consider a single dislocation-disclination line, then Weingarten's theorem says that any displacement discontinuity must be a rigid motion composed of a translation and a rotation. The curvature can then be represented by an integral around a closed curve, deWit (1973), and is composed entirely of rotation terms (angular defects). The dislocation density tensor (the electroweak field) is composed of both translation and rotation terms and we interpret this relationship to imply that the electroweak field is propagated by both massless (translation) and massive charged (rotation) fields.

Curvature and Currents

Similarly, let us uniquely split the second order curvature tensor $L_{\alpha\beta}$ into its symmetric Einstein tensor $G_{\alpha\beta}$ and its antisymmetric components $D_{\alpha\beta}$. From equation (3), we see that the electroweak current is the contraction of this antisymmetric curvature tensor. From the Bianchi Identity (1) we get the symmetric component relation:

$$G_\alpha{}^\beta = -\varepsilon_{0\alpha\gamma\delta}\varepsilon^{0\beta\mu\nu}g_{\nu,\mu}{}^{\delta,\gamma} + [\varepsilon^{0\beta\mu\nu}A_{0\nu\alpha;\mu}]_s \qquad (7)$$

The first term on the right hand side is naturally symmetric and, if expanded, is equivalent to the metric form derived in equation (7.2.6) of Weinberg (1972). His approach is based on the conservation law $G_{\mu\nu;\nu} = 0$ with assumption of small fields, maximally general metric structure, and the nonrelativistic limit giving Newton's law. This agreement, obtained by such different derivations, lends added interest to the symmetric second term $[\varepsilon^{0\beta\mu\nu}A_{0\nu\alpha;\mu}]_s$ which if non-zero implies an electroweak field contribution to Einstein curvature. However, from Maxwell's second equation, $\varepsilon^{0\alpha\mu\nu}F_{\nu\alpha;\mu} = 0$, and together with the symmetry of $W_{\nu\alpha}$, this implies that the electroweak field gives no contribution to the trace $G_\alpha{}^\alpha$. Alternatively, if no evidence is found for an off-diagonal contribution, then

$$[\varepsilon^{0\beta\mu\nu}A_{0\nu\alpha;\mu}]_s = 0 \qquad (8)$$

and this would be a fundamental property of the electroweak field, thereby relating the electromagnetic and weak field gradients. Such a gradient interaction is possible due to the electromagnetic field being a long range force and much stronger than the weak field that acts only over the short range.

If we take the divergence of equation (7), then $G^{\mu\nu}{}_{;\nu} = 0$ as usual. Using the second Bianchi Identity, (2), this implies also that

$$D^{\mu\nu}{}_{;\nu} = 0 \qquad (9)$$

which implies conservation of the antisymmetric curvature as well.

This crack model of currents (curvature) mediated by electroweak fields (dislocations) has a counterpart in special relativistic Feynman diagrams, which can be interpreted as the interaction of two crack tips (currents) through the mediating forces of the dislocation density tensor. The fundamental vertex of the Feynman diagram can thus be visualised as a crack-tip current with its associated fields. This crack model and the Bianchi Identities give us the capability to obtain Feynman diagrams for the more general U_4 space-time.

Conclusion

The crack model is complementary in its results to the unification already performed by gauge approaches. Edelen (1980) has examined the gauge properties of these linear Bianchi equations and argues for a complete analogy with electrodynamics. Parallels exist with Yang-Mills type unified gauge theories and non-Abelian gauge properties of the non-linear Bianchi Identities.

We have not discussed Spin in this crack model; mainly because a number of apparently conflicting approaches already exist in the literature. The most popular approach of Einstein - Cartan - Sciama - Kibble argues that the matter spin tensor is a source of torsion and a corresponding Lagrangian and field equation is hypothesized complementary to the usual one of Einstein, c.f. Hehl et.al. (1976), Kaempffer (1976). We shall not discuss here the field equation for the crack model though a Lagrangian approach to pair production by Rumpf (1979) has shown that a torsion step (gradient) yields fermions. Borschsenius (1976, 1978) has also identified a modified torsion tensor with electroweak fields via Lagrangrian and gauge approaches.

The correspondence of curvature and angular defect has been previously recognised for discrete space-times through Regge calculus. The crack tips (disclinations) are singularities of space-time and it was Einstein and Infeld (1959) who originally treated particles of matter as singularities of the field in otherwise empty space. However, they imposed the singularity on the flat background while the crack model shows it to be a fundamental entity of this more general space-time. In the 3D case, a spherical cavity created in the continuum is represented by the on-diagonal components of the curvature tensor $L_{\alpha\alpha}$. Similarly, mass has been related to holes in spacetime through geometrodynamics, c.f. Misner, Thorne and Wheeler (1973).

We have been able to show with this crack in the Cosmos how the fundamental fields and currents of physics are interwoven. At first, it seems a strange and humorous concept that the material world of particles and fields is a collection of cracks in an otherwise perfectly flat landscape. Yet, the realisation that one's material self is composed of cracks, or defects, is, if not a humbling thought, at least a source of numerous " wisecracks ".

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References

Altamirano, L. and D.Villaroel, 1981, Phys. Rev. D, 24, 12, 3118.

Borchsenuis, K., 1976, Gen. Rel. Grav., 7, 6, 527.

Borchsenius, K., 1978, Nuovo Cimento A, 11, 46, 403.

Davies, J. B. 1980, E.O.S., Trans. Am. Geop. Un., 61, 46, 1051.

deWit, R., 1973, J. Res. N.B.S., A, 1, 77, 49.

Edelen, D.G.B., 1980, Int. J. Eng. Sci., 18, 1095.

Einstein, A., 1956, "The Meaning of Relativity", p.159, Fifth Edition, Princeton University Press, Princeton, N.J.

Einstein, A. and L. Infeld, 1959, Can. J. Math., 1, 209.

Gogala, B., 1980, Int. J. Theor. Phys., 19, 8, 587.

Hehl, F.W., P. Von der Heyde, G.D. Herlick, and J.N. Nester, 1976, Reviews of Modern Physics, 48, 3, 393.

Kaempffer, F.A., 1976, Gen. Rel. Grav., 7, 4, 327.

Lardner, R.W., 1973, Archives of Mechanics, Poland, 25, 6, 911.

Misner, C.W., K.S. Thorne and J.A. Wheeler, 1973, "Gravitation", W.H. Freeman & Co., S.F.

Rauch, R.T., 1982, Phys. Rev. D, 25, 2, 577.

Rumpf, H., 1979, Gen. Rel. Grav., 10, 8, 647.

Schouten, J.A. 1954, "Ricci-Calculus", 2nd edition, Springer-Verlag, Berlin.

Taylor, J.C., 1976, "Gauge theories of weak interactions", Cambridge University Press, Cambridge, U.K.

Weinberg, S., 1972, "Gravitation and Cosmology", John Wiley & Sons, Inc., N.Y.

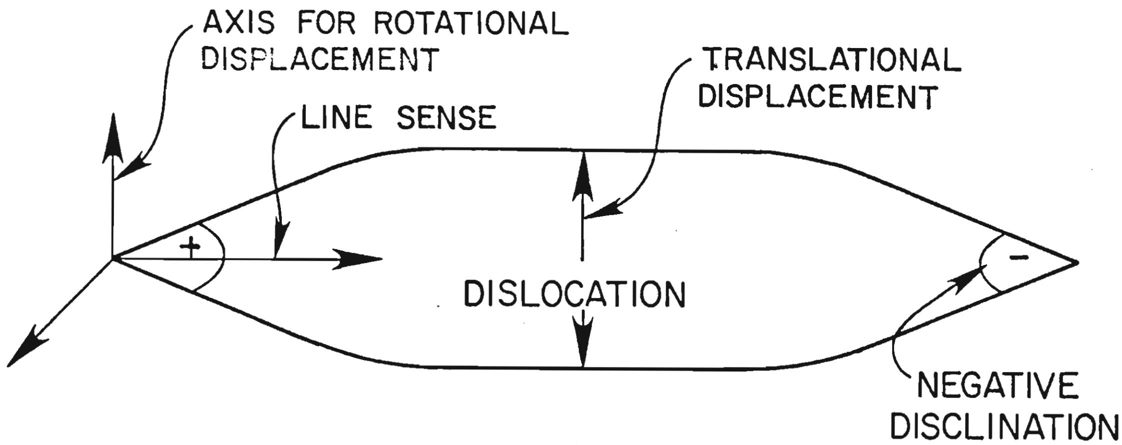
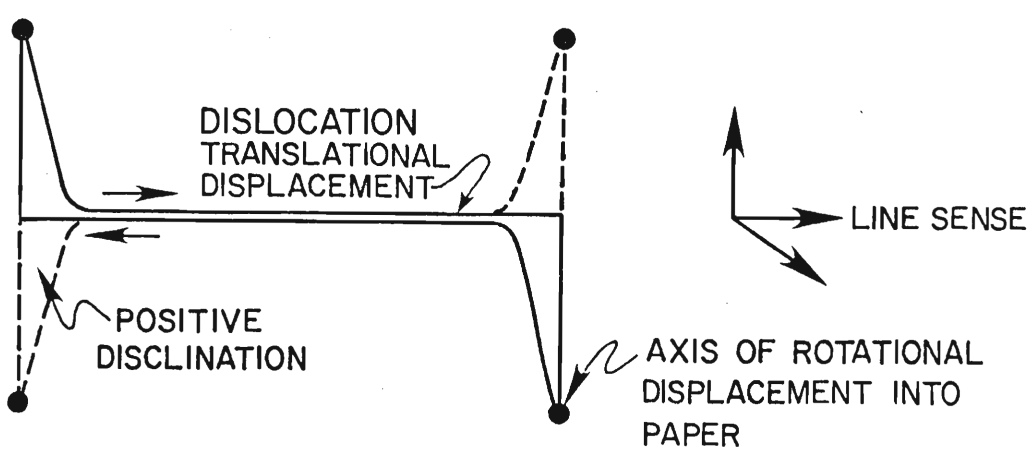


FIGURE 1A

FIGURE 1 illustrates cross-sections of 3D cracks with the location and type of dislocation and disclination. Figure 1A depicts the off-diagonal dislocation density where the displacement vector and line sense are orthogonal. Figure 1B depicts the on-diagonal dislocation density where displacement and line sense are parallel. The solid-line disclination is due to a defect (absence) while the dashed-line disclination is due to an infect (excess).

FIGURE 1B



RESUME

John Bruce Davies

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EDUCATION

B.Sc. 1963, Honours, Physics and Mathematics, University of Wales, Britain.

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Jan. 1978 - June 1978: Discrimination of induced polarisation data and creation of computerised geo-exploration graphics system for Cominco Explorations Ltd.

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1974 - Present: Geo-exploration data collection, analysis and interpretation. Exploration and mining company management and contracting. Magnetic, E-M, Scintillometer, and Electrical surveys.

1973 - 1974: Analysis and Interpretation of seismic array data, administration of research group and design and implementation of analog real-time electronic system of seismogram analysis. Contract with Department of Geophysics, U.B.C.

1970 - 1973: Scientific consultant. Contracts included research, development and application of exploration, data analysis, feedback electronic and software systems.

PUBLICATIONS AND PATENTS

1964 - Patent on system of seismic data analysis.

1968 - "Source Properties of Earthquakes and Discrimination Between Earthquakes and Nuclear Explosions", Bull. Seism. Soc. Am.

1969 - Patent for High-Pressure System Analysis.

1974 - "Analysis of Earthquake Data from McNaughton Reservoir Array", Report to B.C. Dept. of Water and Forests.

1979 - "Maximum Information Universe", Mon. Not. Roy. Ast. Soc.

1979 - Thesis on Fluid Dynamics of Rotating, Gravitating, Dust Gas Systems and Solar System Formation. Asymptotic density solutions, comprised of a central bulge and flat outer disc, were obtained for rotating, self-gravitating discs. Ring instabilities in these dusty gas discs were found to yield planets at locations similar to those in the Solar System. Bode's law behaviour, applicable only in the central bulge, was explained while in the outer disc a constant spacing of planets was predicted as observed.

1980 - "Unique Determination of the Equation of State by Self-Gravitation", Honorable Mention, Gravity Research Foundation Competition.

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