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Maximum information universe

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Summary. The Universe is considered as a photon information channel. The signal is due to galaxies while noise is taken as blackbody thermal radiation. The galaxies are allowed time-dependent luminosities. The effect of Universe expansion on the information capacity of such channels is investigated. A variational approach is used yielding the time dependence of the cosmic scale-factor when the information transmitted to a perfect observer is a maximum. A capacity which is constant and independent of the space-time location of the observer can be considered a Perfect Cosmological Principle. When the observed Deceleration Parameter is taken as 0, 1, 2, the corresponding maximum information universe is an exponential, Page and Milne universe respectively.

Introduction

The object of this research is to investigate the expansion of the Universe and its implications to an observer from the viewpoint of Information Theory. This theory became mathematically established when Shannon (1949) formulated the definition and properties of information channels carrying white-noise data.

Metzner & Morrison (1959) have investigated the rate of flow of information in the various popular cosmological models with specific interest in the dependence of the rate of flow on the distance of the source. They interpreted their results in terms of horizons (Rindler 1956) and the decrease in the rate of transmission of information as such horizons are approached.

The maximum transmission rate of information in a noisy communication channel is defined as the capacity of the channel. This capacity is a function of the spectral powers of the received signal and noise. For photon channels of finite total power, the spectral distribution of the signal in a noiseless channel which yields a maximum capacity is the blackbody thermal distribution (Stern 1960; Bowen 1966).

If we consider the Universe as equivalent to a photon channel, an observer may consider the information signal to be that from the fundamental particles of the Universe – the galaxies. The galaxies will be assumed to behave as blackbodies with time-dependent temperatures. The noise inherent in such a channel may be considered to be that due to the cosmic thermal background radiation of present-time temperature 2.7 K. This approach envisages a perfect observer such that all noise due to the unique space positions of the observer, e.g. interstellar absorption, will be ignored. The effect of intergalactic matter on radiation from galaxies has been found to be negligible to a first approximation (Whitrow & Yallop 1964).

This research will investigate the effect of the Universe expansion on the capacity of such photon channels. A variational approach will be used in order to find the time dependence of the cosmic scale-factor which will yield a maximum information to the perfect observer in a finite time. The luminosity is taken as proportional to an unknown power (n) of the cosmic scale-factor. The observed Deceleration Parameter is then related to n for the cases: 0, 1, 2. These values correspond to exponential, Page and Milne universes respectively, and yield a constant capacity for these photon channels.

A constant information rate to any perfect observer independent of space-time location, can be considered as a Perfect Cosmological Principle. If such a situation exists, then it may well be termed the Equal Opportunity Universe.

Spectral distributions

The Robertson–Walker metric is applicable to all homogeneous isotropic world models:

$$ds^2 = dt^2 - \frac{R^2}{c^2}\{dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\} \Big/ \left(1 + \frac{kr^2}{4}\right)$$

where t is cosmic time, $R(t)$ is the cosmic scale-factor, c is the local velocity of light, r, θ, ϕ are dimensionless spatial coordinates, k is the spatial curvature constant $(+1, 0, -1)$.

Whitrow & Yallop (1963) have shown that the total radiation flux received in a solid angle $d\omega$ in a frequency band $(\nu, \nu - d\nu)$ at time t_0 is

$$I(\nu_0, t_0)\, d\nu_0 d\omega = \frac{n_0}{4\pi}\, c\, d\nu_0 d\omega \int_{t_1}^{t_0} F\left(\frac{\nu_0 R(t_0)}{R(t)}, t\right)\left\{\frac{R(t)}{R(t_0)}\right\}^2 dt \tag{1}$$

where t_1 is the proper time of emission of radiation from the most distant source observed at time t_0 and n_0 is the number of sources per unit proper volume at time t_0. F is the spectral distribution of radiation from a source. It is of interest that this radiation flux is independent of the curvature index k.

If we restrict our observation time to the era of conservation of the total number of galaxies, when Weinberg (1972, p. 414), gives

$$n_0 \propto R(t_0)^{-3}. \tag{2}$$

The galaxies will be considered to be emitting blackbody radiation at the equivalent temperature T. Then

$$F(\nu, t)d\nu = \frac{2\pi h}{c^2} \frac{\nu^3 d\nu}{\{\exp(h\nu/kT) - 1\}}. \tag{3}$$

Thus, the signal flux is

$$S(\nu_0, t_0)\, d\nu_0\, d\omega = K_s\, d\omega\, d_0 R^{-2}(t_0) \int_{t_1}^{t_0} \frac{\nu_0^3 dt}{R(t)\,\{\exp[h\nu_0 R(t_0)/(kRT)] - 1\}} \tag{4}$$

where K_s is constant and we allow the galaxies to have a blackbody temperature which can vary in time.

The noise flux is also a blackbody distribution with the temperature given by (Weinberg 1972, p. 507).

$$\theta(t) \propto R^{-1}(t). \tag{5}$$

Thus, the noise flux is

$$N(\nu_0, t_0)\, d\nu_0 d\omega = K_\mathrm{N} \frac{\nu_0^3 d\nu_0 d\omega}{\{\exp[h\nu_0/k\theta(t_0)] - 1\}} \tag{6}$$

where K_N is constant.

Photon channels

The capacity of photon channels, as a function of bandwidth, frequency distribution of input power and other relevant parameters, has been investigated both by use of the Sampling Theorem (Stern 1960) and use of the Heisenberg Uncertainty Principle (Bowen 1966). The essential property of both approaches is to yield a transmission rate C for a signal power distribution S_ν in a noiseless channel (Bowen 1966). This gives:

$$C \equiv \int_{\nu_0 - B/2}^{\nu_0 + B/2} \mathscr{H}\left(\frac{S_\nu}{h\nu}\right) d\nu \tag{7}$$

where B is the bandwidth, ν_0 is the central frequency and the entropy function is

$$\mathscr{H}(x) \equiv (1 + x) \ln (1 + x) - x \ln x. \tag{8}$$

When the channel has signal and noise as independent processes, the transmission rate of information becomes (Stern 1960),

$$C = \int_{\nu_0 - B/2}^{\nu_0 + B/2} \left\{\mathscr{H}\left(\frac{S_\nu + N_\nu}{h\nu}\right) - \mathscr{H}\left(\frac{N_\nu}{h\nu}\right)\right\} d\nu. \tag{9}$$

In the case where both the signal and the noise powers are sufficiently large the capacity is,

$$C \simeq B\left\{\ln\left(\frac{S + N}{h\nu_0 B}\right) - \ln\left(\frac{N}{h\nu_0 B}\right)\right\} \tag{10}$$

for the conditions

$$N \gg h\nu_0 B, \quad S \gg h\nu_0 B.$$

Thus

$$C \simeq B \cdot \ln\left\{\frac{S + N}{N}\right\} \tag{11}$$

where

$$S = \int_{\nu_0 - B/2}^{\nu_0 + B/2} S_\nu d\nu, \quad N = \int_{\nu_0 - B/2}^{\nu_0 + B/2} N_\nu d\nu.$$

Universe as a photon channel

When we consider the signal in this noisy photon channel as due to galaxies, and the noise as due to thermal microwave radiation, the approximation considered above becomes valid under present time conditions. Thus

$$S(t_0)=K_s\int_{\nu_0-B/2}^{\nu_0+B/2}\nu_0^3 d\nu_0\int_{t_1}^{t_0}\frac{R^{-2}(t_0)}{R(t)}\frac{dt}{\{\exp[h\nu R(t_0)/kT(t)R(t)]-1\}}. \tag{12}$$

Putting $\nu_0=B/2$ and allowing B to be large enough to enclose all frequencies of interest, $B\to\infty$, we get by interchanging the order of integration,

$$S(t_0)=K_1R^{-2}(t_0)\int_{t_1}^{t_0}\frac{T^4(t)}{R(t)}\left\{\frac{R(t)}{R(t_0)}\right\}^4 dt$$

and

$$N(t_0)=K_2R^{-4}(t_0).$$

Thus as K_1, K_2 are constants

$$C=B\ln\{1+S/N\}$$

we have

$$C=B\ln\left\{1+K_{12}R^{-2}(t_0)\int_{t_1}^{t_0}R^3(t)T^4(t)\,dt\right\} \tag{13}$$

where K_{12} is a constant.

This expression for the rate of information transmission to a perfect observer indicates that the dependence on time of this capacity is implicit through the time dependence of the cosmic scale-factor R and the mean galaxy temperature T.

We can now formulate mathematically the question: what is the time dependence of the cosmic scale-factor $R(t)$ which will yield a maximum information to the perfect observer over a finite time?

The total information received is

$$I_t=\int_{t_b}^{t_d}C(t_0)\,dt_0 \tag{14}$$

where t_b = time of observer birth, t_d = time of observer death.

This is a problem of the calculus of variations and we can use Euler's equations to obtain a differential equation governing R, T.

Thus

$$I_t=B\int_{t_b}^{t_d}dt_0\ln\left\{1+K_{12}R^{-2}(t_0)\int_{t_1}^{t_0}R^3(t)\,T^4(t)\,dt\right\}. \tag{15}$$

If we put

$$g(t_0)=\int_{t_1}^{t_0}R^3T^4dt \tag{16a}$$

then

$$\frac{dg}{dt_0}=R^3(t_0)\,T^4(t_0) \tag{16b}$$

where we put $t_1=0$ such that time is measured relative to t_1.

Now consider the case where the fourth power of the blackbody temperature, and thus the luminosity of galaxies, is proportional to some power n of the cosmic scale-factor, R.

$$T^4 \propto R^n. \tag{17}$$

Thus we wish to maximize,

$$I_t = B \int_{t_b}^{t_d} dt_0 \ln \left\{1 + K'_{12} R^{-2}(t_0) \int_{t_1=0}^{t_0} R^{3+n}(t)\, dt\right\}. \tag{18}$$

Using the Lagrange Undetermined Multiplier $\lambda(t_0)$ this is equivalent to

$$\delta I_t \equiv \delta \int_{t_b}^{t_d} dt_0 \{B \ln [1 + K'_{12} R^{-2}(t_0) g(t_0)]\} + \lambda(t_0)\left[\frac{dg}{dt_0} - R^{3+n}(t_0)\right] = 0.$$

Variation of the Total Information I_t is obtained by use of Euler's equations. For R we have

$$\frac{\partial I_t}{\partial R} = 0$$

thus

$$\lambda = \frac{-2K'_{12}}{(3+n)} \{R^{-5-n} g/(1 + K'_{12} R^{-2} g)\}. \tag{19}$$

Euler's equation for g is

$$\frac{\partial I_t}{\partial g} - \frac{d}{dt_0} \frac{\partial I_t}{\partial g} = 0$$

which yields

$$\frac{d\lambda}{dt_0} = K'_{12} R^{-2}/(1 + K'_{12} R^{-2} g). \tag{20}$$

Combining (19) and (20), in order to remove the term in λ, gives:

$$(5+n)\left\{1 - 2R^{-4-n} \frac{dR}{dt_0} g\right\} = -K'_{12}(3+n) R^{-2} \left\{1 - 2R^{-4-n} \frac{dR}{dt_0} g\right\}. \tag{21}$$

An exact solution to this equation is obviously

$$2g \frac{dR}{dt_0} = R^{4+n}. \tag{22}$$

Using

$$\frac{dg}{dt_0} = R^{3+n}$$

yields

$$(2+n)\left(\frac{dR}{dt_0}\right)^2 = R \frac{d^2R}{dt_0^2}. \tag{23}$$

Integrating once yields

$$\frac{dR}{dt_0} = bR^{2+n} \tag{24}$$

where b is the integration constant. Integrating once more gives

$$R^{1+n} = \frac{-1}{(1+n)(c+bt)}, \quad n \neq -1 \tag{25}$$

and

$$R = c \exp(bt), \quad n = -1$$

where c is an integration constant. From equation (18), it can readily be seen that for these exponential ($n = -1$) solutions the capacity is constant and independent of the space-time location of the perfect observer.

Implications

Thus we will obtain different time dependences of the cosmic scale-factor when the temperature of galaxies is not kept constant. The Deceleration Parameter, q_0 is defined as

$$q_0 \equiv -R \frac{d^2R}{dt_0^2} \bigg/ \left(\frac{dR}{dt_0}\right)^2 = -(n+2) \tag{26}$$

and Hubble's Parameter, H_0, as

$$H_0 \equiv \frac{dR}{dt_0} \bigg/ R = bR^{1+n}. \tag{27}$$

However, galactic evolution will affect the Deceleration Parameter, such that the observed Deceleration Parameter, q_{obs}, will be related to the true q_0 by (Weinberg 1972, p. 444)

$$q_{\text{obs}} = q_0 - E_0/H_0 \tag{28}$$

where

$$E_0 \equiv \frac{dL}{dt_0} \bigg/ L$$

where L is the luminosity. Assuming blackbody radiation, we have

$$L \propto T^4 \propto R^n$$

which yields

$$\frac{dL}{dt_0} \bigg/ L = nH_0; \quad q_{\text{obs}} = q_0 - n. \tag{29}$$

Present observational evidence indicates that q_{obs} has the value 1 ± 1. Let us look at the implications of taking the observed Deceleration Parameter to be 0, 1 or 2.

Case 1: $q_{\text{obs}} = 0$.

In this case, q_0 is given by n and thus $n = -1$. We have:

$$R = c \exp(bt); \quad T \propto \exp(-bt/4); \quad \theta \propto \exp(-bt).$$

The capacity, i.e. information rate, is constant in this exponential universe. As b is an integration constant, this may be an exponentially expanding, contracting or oscillating universe.

Case 2: $q_{\mathrm{obs}} = 1$.

Here we find $n = -1.5$

$$R = 4(c + bt)^2; \quad T \propto (c + bt)^{-3/4}; \quad \theta \propto (c + bt)^{-2}.$$

This is similar to a Page universe where $R \propto t^2$. The information rate in such a universe is constant in time when c is zero.

Case 3: $q_{\mathrm{obs}} = 2$.

Here we get $n = -2$

$$R = c + bt; \quad T \propto (c + bt)^{-1/2}; \quad \theta \propto (c + bt)^{-1}.$$

This is similar to a Milne universe where $R \propto t$. The information rate in such a universe is constant when c is taken to be zero.

In all these cases, when $\mathrm{Re}(b) > 0$, we find that the temperature and thus the luminosity of galaxies decreases with age. Thus as we observe galaxies with increasing redshifts, we would expect higher luminosities, a result which observations confirm. Weedman (1976) has argued that galaxies, Seyfert galaxies and quasars, are a single family of objects with increasing luminosity as a function of cosmological redshift.

An interesting consequence of this information theory approach is that when the information rate for all observers is constant in these universes, such a universe can be considered to obey a Perfect Cosmological Principle. In effect, this Perfect Cosmological Principle, i.e. the information rate to all observers is constant and independent of space-time location, can be considered the manifesto of the Equal Opportunity universe.

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